CROSS DERIVATIVE UPDATE IN VOLUME ELEMENTS

CMISS Document No.: 2002/003
Authour : Kumar Mithraratne
Date : October 2002.

1. INTRODUCTION

This document describes the update of cross derivatives in a volume mesh (initially linear). A volume mesh consisting of tri-cubic elements, if collapsed nodes are not present, have 8 corner nodes which are attached only to one element as shown in the figure 1. In addition to this, there are nodes on the edges, each of these are shared by two elements, nodes on the faces shared by four elements and the interior nodes which are shared by eight elements.

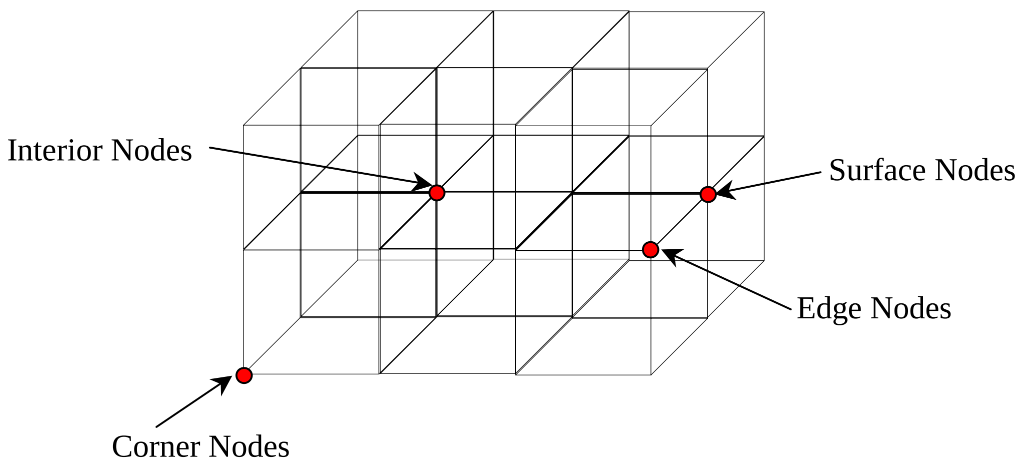
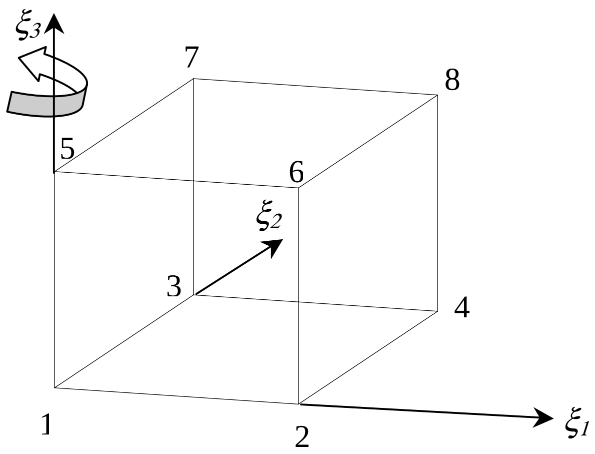


Figure 1.1 Nodal locations in a volume mesh

2. CORNER NODES



Face 1 (ξ_2 & ξ_3) :	1 3 5 7
Face 2 (ξ_2 & ξ_3) :	2 4 6 8
Face 3 (ξ_3 & ξ_1) :	1 5 2 6
Face 4 (ξ_3 & ξ_1) :	3 7 4 8
Face 5 (ξ_1 & ξ_2) :	1 2 3 4
Face 6 (ξ_1 & ξ_2) :	5 6 7 8

Figure 2.1 Nodes and associated faces in a volume element.

Node 1:

The second derivative of a dependent variable can be expanded in the following manner,

$$\left(\frac{\partial^2 U}{\partial S_1.\partial S_2}\right)_1 = \frac{\left(\frac{\partial U}{\partial S_1}\right)_{\xi_2=1.0} - \left(\frac{\partial U}{\partial S_1}\right)_{\xi_2=0.0}}{\Delta S_2} = \frac{\frac{\left(U_{\xi_1=1.0} - U_{\xi_1=0.0}\right)_{\xi_2=1.0}}{\Delta S_1} - \frac{\left(U_{\xi_1=1.0} - U_{\xi_1=0.0}\right)_{\xi_2=0.0}}{\Delta S_1}}{\Delta S_2}$$

Where ΔS_1 and ΔS_2 represent the arc length in ξ_1 and ξ_2 directions respectively.

$$\left(\frac{\partial^2 U}{\partial S_1.\partial S_2}\right)_1 = \frac{U_{\xi_1=1.0,\xi_2=1.0} + U_{\xi_1=0.0,\xi_2=1.0} - U_{\xi_1=1.0,\xi_2=0.0} - U_{\xi_1=0.0,\xi_2=0.0}}{\Delta S_1.\Delta S_2} = \frac{U_1 + U_4 - U_2 - U_3}{\Delta S_1.\Delta S_2} \qquad (2.1)$$

In equation (2.1), the product $\Delta S_1.\Delta S_2$ can however be better represented by the area of the face (in this case face no. 5 – see figure 2.1) as all four nodes of the face are involved in the second derivative.

Face 5 : $$\left(\frac{\partial^2 U}{\partial S_1.\partial S_2}\right)_1 = \frac{U_1 + U_4 - U_2 - U_3}{A_{F5}}$$

Face 1: $$\left(\frac{\partial^2 U}{\partial S_2.\partial S_3}\right)_1 = \frac{U_1 + U_7 - U_3 - U_5}{A_{F1}}$$

Face 3: $$\left(\frac{\partial^2 U}{\partial S_3.\partial S_1}\right)_1 = \frac{U_1 + U_6 - U_2 - U_5}{A_{F3}}$$

We can thus write similar expressions for the second derivatives involved with the other corner nodes as follows.

Node 2:

Face 5: $$\left(\frac{\partial^2 U}{\partial S_1.\partial S_2}\right)_2 = \frac{U_1 + U_4 - U_2 - U_3}{A_{F5}}$$

Face 2: $$\left(\frac{\partial^2 U}{\partial S_2.\partial S_3}\right)_2 = \frac{U_2 + U_8 - U_4 - U_6}{A_{F2}}$$

Face 3: $$\left(\frac{\partial^2 U}{\partial S_3.\partial S_1}\right)_2 = \frac{U_1 + U_6 - U_2 - U_5}{A_{F3}}$$

Node 3:

Face 5: $$\left(\frac{\partial^2 U}{\partial S_1.\partial S_2}\right)_3 = \frac{U_1 + U_4 - U_2 - U_3}{A_{F5}}$$

Face 1: $$\left(\frac{\partial^2 U}{\partial S_2.\partial S_3}\right)_3 = \frac{U_1 + U_7 - U_3 - U_5}{A_{F1}}$$

Face 4: $$\left(\frac{\partial^2 U}{\partial S_3.\partial S_1}\right)_3 = \frac{U_3 + U_8 - U_4 - U_7}{A_{F4}}$$

Node 4 :

Face 5: $$\left(\frac{\partial^2 U}{\partial S_1.\partial S_2}\right)_4 = \frac{U_1 + U_4 - U_2 - U_3}{A_{F5}}$$

Face 2: $$\left(\frac{\partial^2 U}{\partial S_2.\partial S_3}\right)_4 = \frac{U_2 + U_8 - U_4 - U_6}{A_{F2}}$$

Face 4: $$\left(\frac{\partial^2 U}{\partial S_3.\partial S_1}\right)_4 = \frac{U_3 + U_8 - U_4 - U_7}{A_{F4}}$$

Node 5 :

Face 6: $$\left(\frac{\partial^2 U}{\partial S_1.\partial S_2}\right)_5 = \frac{U_5 + U_8 - U_6 - U_7}{A_{F6}}$$

Face 1: $$\left(\frac{\partial^2 U}{\partial S_2.\partial S_3}\right)_5 = \frac{U_1 + U_7 - U_3 - U_5}{A_{F1}}$$

Face 3: $$\left(\frac{\partial^2 U}{\partial S_3.\partial S_1}\right)_5 = \frac{U_1 + U_6 - U_2 - U_5}{A_{F3}}$$

Node 6 :

Face 6: $$\left(\frac{\partial^2 U}{\partial S_1.\partial S_2}\right)_6 = \frac{U_5 + U_8 - U_6 - U_7}{A_{F6}}$$

Face 2: $$\left(\frac{\partial^2 U}{\partial S_2.\partial S_3}\right)_6 = \frac{U_2 + U_8 - U_4 - U_6}{A_{F2}}$$

Face 3: $$\left(\frac{\partial^2 U}{\partial S_3.\partial S_1}\right)_6 = \frac{U_1 + U_6 - U_2 - U_5}{A_{F3}}$$

Node 7 :

Face 6: $$\left(\frac{\partial^2 U}{\partial S_1.\partial S_2}\right)_7 = \frac{U_5 + U_8 - U_6 - U_7}{A_{F6}}$$

Face 1: $$\left(\frac{\partial^2 U}{\partial S_2.\partial S_3}\right)_7 = \frac{U_1 + U_7 - U_3 - U_5}{A_{F1}}$$

Face 4: $$\left(\frac{\partial^2 U}{\partial S_3.\partial S_1}\right)_7 = \frac{U_3 + U_8 - U_4 - U_7}{A_{F4}}$$

Node 8 :

Face 6: $$\left(\frac{\partial^2 U}{\partial S_1.\partial S_2}\right)_8 = \frac{U_5 + U_8 - U_6 - U_7}{A_{F6}}$$

Face 2: $$\left(\frac{\partial^2 U}{\partial S_2.\partial S_3}\right)_8 = \frac{U_2 + U_8 - U_4 - U_6}{A_{F2}}$$

Face 4: $$\left(\frac{\partial^2 U}{\partial S_3.\partial S_1}\right)_8 = \frac{U_3 + U_8 - U_4 - U_7}{A_{F4}}$$

3. EDGE NODES

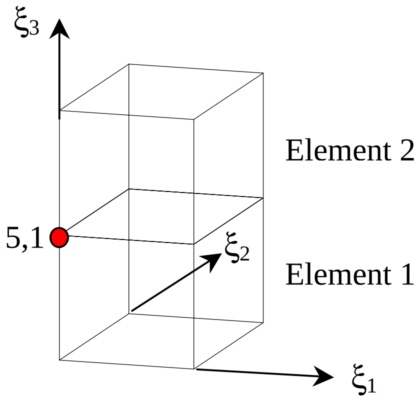


Figure 3.1 An edge node shared by two adjacent elements.

As was shown earlier, these are shared by two elements in the mesh. It is therefore necessary to consider the contributions from both the elements. For instance, if we consider the node in the mesh shown in figure 3.1, the local node number with respect to the element 1 is 5 and that with respect to the element number 2 is 1. The cross derivative at the node 5 is therefore determined using the equations under node 5 and those under node 1 in the section 3 and then taking the average of both.

4. SURFACE AND INTERIOR NODES

The method described in the section 3 can be extended to determine the cross-derivatives of the nodes shared by four elements (surface) and eight elements (interior).